

Lecture 10/17/23: Polynomial Functions

Defn: A polynomial function is a function that can be written as

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

where a_0, a_1, \dots, a_n are ~~numbers~~ #s with $a_n \neq 0$.

$n, n-1, \dots, 2, 1$ are whole pos. #s.

• The degree of $p(x)$ is highest power of x

• leading term is $a_n x^n$

• leading coeff. is a_n .

$$\text{Deg}(f \cdot g) = \text{deg}(f) + \text{deg}(g)$$

Ex: which are polynomials

a) $f(s) = 5s + 7$ Yes

b) $2x^2 + x^{1.5} + x$ No!
 ← can't have 1.5

c) $2 + t^{12} + t^4$ Yes!

Ex: what is the degree and # of nonzero terms and leading coefficient of

$$r(x) = x^2 - 7x^3 + 2x^4 + 1 ; h(x) = (x+4)(x-3)(2x+1)$$

degree $(r(x)) = 4$; 4 nonzero terms ; leading term $2x^4$.

degree $(h(x)) = 3$; 4 nonzero terms ; leading term $2x^3$.

Longrun Behavior of Pdots

The long run behavior of a polynomial $a_n x^n + \dots + a_1 x + a_0$ is the long run behavior of its leading term $a_n x^n$.

Ex: What is the long run behaviour of

$$3.7x^3 + 12x - 2x^6$$

↑
Leading term!

comes from
- coefficient.

$$\text{As } x \rightarrow \pm\infty \quad y \rightarrow -\infty$$

- #7
- a) Even no
Odd Yes
 - b) Even Yes and neg. coef.
Odd No
 - c) Even Yes ~~odd~~ ^{neg} coeff.
Odd Yes ~~odd~~ neg. coeff.

